obtained by the above procedure, showed the same results as those of Bothe (*Zeit. für Naturforschung*, 4, 542 (1949)). The peak positions of the spectra were as follows: for Al, 0.67; Cu, 0.87; Sn, 0.91; Pb, 0.93, where the unit was $E/E_0$ and $E_0$ was 4833 $H\rho$.

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## 2. Trial Construction of the Small Betatron. (I)

### Pole Face Angle Determination for the Focusing Magnetic Field

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Magnetic field which decreases according to the increase of radius is always necessary in circular accelerators and magnet type beta-ray spectrometer, where the magnetic focusing is of importance. It is difficult to solve this problem analytically, and few results have been reported. But the following simple consideration can give the result which is in fairly good agreement with some experimental results.

We assume first, that the pole faces of the magnet are the magnetic equipotential surfaces and the line of force is of such a shape as indicated in Fig. 1. Next, we assume that the magnetic field decreases with increasing radius in the range of $r=r_1$

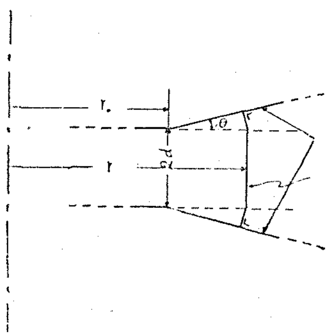


Fig. 1. Assumed shapes of the magnet and magnetic line of force.

to $r=r_2$ in such a manner as is required from the magnetic focusing condition.

Now the magnetic field strength $H$ at radius $r$ from the center is given by

$$H=\frac{V}{l},$$

where $V$ is the magnetic potential difference between poles and $l$ is the length of the line of force which is given by

$$t=2\ \{d+(r-r_0)\theta\}.$$

Then $\theta$ satisfies the following equation,

$$\theta=-\{d+(r-r_0)\theta\}\frac{1}{H}\ \frac{dH}{dr}. \qquad (1)^*$$

In the case of betatron, it is required that $H$ varies as $r^{-n}$. In this case, the right hand side of Eq. (1) shall be averaged over the range from $r=r_1$ to $r=r_2$. Simple calculation shows that

$$\theta=\frac{\dfrac{nd}{r_2-r_1}\ \mathrm{In}\ \dfrac{r_2}{r_1}}{1-n\ (1-\dfrac{r_0}{r_2-r_1}\ \mathrm{In}\ \dfrac{r_2}{r_1})} \qquad (2)$$

Results estimated by Eq. (2) are compared with some known experimental values of $\theta$ in the following table.

Table 1.

| Reference | $n$ | $d$ | $r_0$ | $r_1$ | $r_2$ | $\theta$ calc. | $\theta$ experiment |
|---|---|---|---|---|---|---|---|
| | | inches | | | | | |
| Westendorp and Charlton [1] | 0.75 | 2.87 | 27.944 | 32.0 | 35.5 | 4°12′ | 4°11′ |
| Kerst *et al.* [2] | 0.5 | 0.649 | 9.403 | 9.64 | 10.43 | 1°38′ | 1°15′ |
| Authors [3] | 0.75 | 0.516 | 1.18 | 2.36 | 3.54 | 13°53′ | 14° |

* Similar result was obtained by R. R. Wilson: *J. App. Phys.* **11**, 781 (1940), but his result differed by the factor 2 from the Eq. (1).

(1) W. F. Westendorp and E. E. Charlston: *J. App. Phys.* **16**, 581(1945).

(2) D. W. Kerst, G. D. Adams, H. W. Koch and C. S. Robinson. *R. S. I.* **12**, 462 (1950).

(3) Now under construction. Similar to that of Kerst's prior work. D. W. Kerst: *Phys. Rev.* **60**, 47 (1941).